Computer Programming

Tail recursion. Decision. Assignment. Iteration

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Two ways of writing recursion

```
unsigned max(unsigned a, unsigned b) { return a > b ? a : b; }

unsigned maxdig(unsigned n) {
  return n < 10 ? n : max(n%10, maxdig(n/10));
} // directly from: number ::= digit | number digit

unsigned maxdig2(unsigned n, unsigned maxd) {
  unsigned md1 = max(n%10, maxd);
  return n < 10 ? md1 : maxdig2(n/10, md1);
} // keep maxd found so far

unsigned maxdig1(unsigned n) {
  return n < 10 ? n : maxdig2(n/10, n%10);
} // 1-arg wrapper for function above
```

Is recursion efficient?

$S_0 = 1, \quad S_n = S_{n-1} + \cos n \qquad S_{1000000} = ?$

```
#include <stdio.h>
#include <math.h>

double s(unsigned n) {
  return n == 0 ? 1 : s(n-1) + cos(n);
}

int main(void) {
  printf("%f\n", s(1000000));
  return 0;
}
```

```
./a.out
Segmentation fault
```

Recursion and the stack

Code executes sequentially (except for branch/call/return)

When calling a function, must remember where to return
(right after call)

Must remember function parameters and locals to keep using them

These are placed on the *stack*
since nested calls return in opposite order made
must restore values in reverse order of saving (last in, first out)

If recursion is very deep, stack may be insufficient $\Rightarrow$ program crash
even if not, save/call/restore may be expensive

Tail recursion

$S_0 = 1, \quad S_n = S_{n-1} + \cos n$

We *know* we'll have to add $\cos n$ (but not yet to what)

$\Rightarrow$ can *anticipate* and *accumulate* values we need to add

When reaching the base case, add accumulator (partial result)

```
double s2(double acc, unsigned n)
{
  return n == 0 ? 1 + acc : s2(acc + cos(n), n-1);
}

double s1(unsigned n) { return s2(0, n); }
```

Program now works!

Tail recursion IS iteration!

A function is *tail-recursive* if recursive call is *last* in the function.
no computation done after call (e.g., with result)
result (if any) is returned unchanged between calls

$\Rightarrow$ parameter and local values no longer needed
$\Rightarrow$ *no need for stack*: replace *recursive* call with jump,
return value at end (base case)

(Optimizing) compiler converts tail recursion to iteration (loop)
need not worry about efficiency

Review: conditional expression

condition ? *expr1* : *expr2* — everything is an *expression*

expr1 or *expr2* may be conditional expression themselves
(if we need more questions to find out the answer)

$$f(x) = \begin{cases} -6 & x < -3 \\ 2 \cdot x & x \in [-3, 3] \\ 6 & x > 3 \end{cases}$$

```
double f(double x)
{
  return x < -3 ? -6      // else, we know x >= -3
                : x <= 3 ? 2*x : 6;
}
```

or: `x >= -3 ? (x <= 3 ? 2*x : 6) : -6`

if $x \geq -3$ we still need to ask $x \leq 3$?

or: `x < -3 ? -6 : (x > 3 ? 6 : 2*x)`

if x is not < -3 or > 3, it must be $x \in [-3, 3]$

Conditional expression (cont'd)

The conditional expression is an expression

⇒ may be used *anywhere* an expression is needed

Example: as an expression of type string in `puts`

(function that prints a string to stdout, followed by a newline)

```
#include <ctype.h>
#include <stdio.h>

int main(void)
{
  int c = getchar();  // variable initialized w/ char read
  puts(isupper(c) ? "uppercase letter"
         : islower(c) ? "lowercase letter"
         : isdigit(c) ? "digit"
         : "not letter or digit");
  return 0;
}
```

Note layout for readability: one question per line.

Expressions and statements

Expression: *computes a result*

arithmetic operations: `x + 1`

function call: `fact(5)`

Statement: *executes an action*

`return n + 1;`

Any *expression* followed by `;` becomes a *statement*

`n + 3;` (computes, but does not use the result)

`printf("hello!");` we do not use the *result* of `printf`

but are interested in the *side effect*, printing

`printf` returns an `int`: number of chars written (rarely used)

Statements contain expressions. Expressions don't contain statements.

Sequencing

Statements are written and executed in order (*sequentially*)

With *decision*, *recursion* and *sequencing* we can write any program

Compound statement: several statements between *braces* `{ }`
A *function body* is a compound statement (*block*).

```
{                    {
    statement          int c = getchar();
    ...                printf("let's print the char: ");
    statement          putchar(c);
}                    }
```

A compound statement is considered *a single statement.*
May contain declarations: anywhere (C99/C11)/at start (C89).
All other statements are *terminated* by a semicolon `;`

The *sequencing operator* is the *comma*: *expr1* `,` *expr2*
Evaluate *expr1*, ignore, the value of the expression is that of *expr2*

The conditional statement (`if`)

Conditional operator `?` `:` selects from two *expressions* to evaluate

Conditional statement selects between two *statements* to execute

Syntax:

`if` `(` *expression* `)`
  statement1
`else`
  statement2

or

`if` `(` *expression* `)`
  statement1

Effect:

If the expresson is *true* (nonzero) *statement1* is executed,
else *statement2* is executed (or nothing, if the latter is missing)

Each branch has only *one* statement. If several statements are needed, these must be grouped in a *compound statement* `{ }`

The *parantheses* `( )` around the condition are mandatory.
The `else` branch always belongs to the *closest* `if` :

```
if (x > 0) if (y > 0) printf("x+, y+"); else printf("x+, y-");
```

Example with the `if` statement

Printing roots of a quadratic equation:

```
void printsol(double a, double b, double c)
{
  double delta = b * b - 4 *a * c;
  if (delta >= 0) {
    printf("root 1: %f\n", (-b-sqrt(delta))/2/a);
    printf("root 2: %f\n", (-b+sqrt(delta))/2/a);
  } else printf("no solution\n"); // puts("no solution");
}
```

Can rewrite the *conditional operator* `? :` using the `if` *statement*

```
int abs(int x)
{
  return x > 0 ? x : -x;
}
```

```
int abs(int x)
{
  if (x > 0) return x;
  else return -x;
}
```

Decisions with multiple branches

The branches of an `if` can be any statements

⇒ also `if` statements

⇒ can chain decisions one after another

```
// ints a and b have been read before
int c = getchar(); // read operator
if (c == '+') return a + b;
else if (c == '-') return a - b;
else { puts("bad operator"); exit(1); } // exit program
```

The checks `c=='+'` and `c=='-'` are *not independent*. *DON'T* write

~~`if (c == '+') return a + b;`~~

~~`if (c == '-') return a - b;`~~

It is pointless do the second test if the first was true

(c cannot be both + and −)

The proper code is with chained `if`s (or a `switch` statement)

Example with `if`: printing a number

```c
#include <stdio.h>

void printnat(unsigned n) { // recursive, digit by digit
  if (n >= 10)              // if it has several digits
    printnat(n/10);         // write first part
  putchar('0' + n % 10); // always write last digit
}

int main(void)
{
  printnat(312);
  return 0;
}
```

Logical expressions in C

The *condition* in the `if` statement or the `? :` operator is usually a *relational expression*, with a *logical value*: `x != 0`, `n < 5`, etc.
The C language was conceived without a special boolean type (since C99, `stdbool.h` has `bool`, `false` (0) and `true` (1)).

A value is considered *true* when *nonzero* and *false* when *zero*
(when used as a condition in `? :` , `if` , `while` etc.)
⇒ condition must have *scalar* type (integer, floating point, enum)

Comparison operators (`==` `!=` `<` etc.)
return the *integer* values 1 (for *true*) or 0 (for *false*)
⇒ suitable for direct use as conditions

Library functions often return zero or nonzero (NOT zero or one!)
only compare `if (isdigit(c))` (nonzero), don't compare to 1 !

Logical operators

With logical operators, we can write complex decisions:

expr	! *expr*
0	1
$\neq 0$	0

negation ! NOT

e_1 && e_2	e_2: 0	e_2: $\neq 0$
e_1: 0	0	0
e_1: $\neq 0$	0	1

conjunction && AND

e_1 || e_2	e_2: 0	e_2: $\neq 0$
e_1: 0	0	1
e_1: $\neq 0$	1	1

disjunction || OR

Reminder: logical operators produce 1 for *true*, 0 for *false*
An integer is interpreted as *true* if *nonzero*, and as *false* if 0

Example: leap year

A year is a leap year if

it is divisible by 4 **and**
it is **not** divisible by 100 **or** it is divisible by 400

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```
int isleap(unsigned yr)     // 1: leap year, 0: not
{
  return yr % 4 == 0 && (!(yr % 100 == 0) || yr % 400 == 0);
}
```

`!(yr % 100 == 0)` is equivalent with `(yr % 100 != 0)`

Precedence of logical operators

The *unary logical operator* `!` (logical negation): highest precedence
`if (!found)` same as `if (found == 0)` (zero is false)
`if (found)` same as `if (found != 0)` (nonzero is true)

Relational operators: lower precedence than arithmetic ones
$\Rightarrow$ we can naturally write `x < y + 1` for `x < (y + 1)`
Precedence: `>` `>=` `<` `<=` , then `==` `!=`

Binary logic operators: `&&` (AND) evaluated before `||` (OR)
have lower precedence than relational operators
$\Rightarrow$ can naturally write `x < y + z && y < z + x`

Short-circuit evaluation

Logical expressions are evaluated *left to right*
(in general, for other operators, evaluation order is *unspecified*)
Evaluation stops (*short-circuit*) when the result is known:
for `&&`, when the left argument is false (right is not evaluated)
for `||`, when the left argument is true

```
if (p != 0 && n % p == 0)
  printf("p divides n");

if (p != 0)                 // only if nonzero
  if  (n % p == 0)          // test the remainder
    printf("p divides n");
```

⇒ Be careful when writing compound tests!
⇒ Avoid side-effects in compound tests (or place them first)

Evaluation order and precedence are different notions!
`2 * f(x) + g(x)` : multiplication before addition (precedence)
Unspecified which part of sum is *evaluated* first (`f` or `g`)

Assignment

In recursive functions we don't need to change variable values
  a programming style typical for (pure) *functional languages*
Recursive calls create *new parameter instances* with *new values*.

In *imperative programming*, we use:
variables to represent objects used in solving the problem
  (current character; partial result; number left to process)
assignment, to give a *new value* to a variable
  (to express a computation step in the program)

Syntax: *variable* = *expression*
Everything is an *assignment expression*.

Effect: 1. The expression is evaluated
2. the value is *assigned* to the variable and becomes the value of the entire expression.
Example: `c = getchar()` `n = n-1` `r = r * n`

Assignment (cont'd)

May appear in other expressions: `if ((c = getchar()) != EOF)`
...
May be chained: `a = b = x+3` (`a` and `b` get the same value)

Any *expression* (function call, assignment) with `;` is a *statement*
`printf("hello");` `c = getchar();` `x = x + 1;`

A variable changes value *only by assignment*!
NOT in other expressions, or by passing as parameter!
`n + 1` `sqr(x)` `toupper(c)` compute, DON'T change!
`n = n + 1` `x = sqr(x)` `c = toupper(c)` *change*

WARNING! `=` assignment `==` comparison.

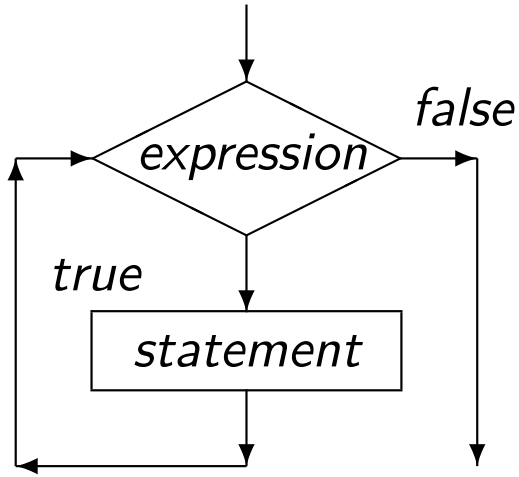
Iteration. The `while` loop (initial test)

Expresses the repetition of a statement, guarded by a condition:

Syntax:

```
while ( expression )
   statement
```

!!! Expression must be between parantheses ( )



Semantics: evaluate expression. If it is true (nonzero):

(1) execute statement (loop *body*)

(2) go back to start of `while` (evaluate expression)

Else (if condition is false/zero), don't execute anything.

$\Rightarrow$ body executes repeatedly, as long as (while) condition is true

Iteration and recursion

We can define iteration (the while loop) recursively:

```
while ( expression )
   statement
```

is the same as

```
if ( expression ) {
   statement
   while ( expression )
      statement
}
```

Rewriting recursion as iteration

```
unsigned fact_r(unsigned n,
                unsigned r) {
  return n > 0
      ? fact_r(n - 1, r * n)
      : r;
}
// called with fact_r(n, 1)
```

```
unsigned fact_it(unsigned n) {
  unsigned r = 1;
  while (n > 0) {
    r = r * n;
    n = n - 1;
  }
  return r;
}
```

```
int pow_r(int x, unsigned n,
                 int r) {
  return n > 0
       ? pow_r(x, n-1, x*r)
       : r;
}
// called with pow_r(x, n, 1)
```

```
int pow_it(int x, unsigned n) {
  int r = 1;
  while (n > 0) {
    r = x * r;
    n = n - 1;
  }
  return r;
}
```

Rewriting recursion as iteration

Easier if function is written by accumulating a partial result (*tail recursion*)

Stop test and initial result value are the same as in recursion

Recursion creates *new instances* of parameters for each recursive call, with new values dependent on the old ones:

ex. `n * r`, `n - 1`, `x * r`, etc.

Iteration *updates (assigns)* values to variables in each iteration, using the same rules/expressions

Ex. `r = n * r`, `n = n - 1`, `r = x * r`

Both variants return the accumulated result

!!!: Recursion and iteration both repeat a processing step
$\Rightarrow$ in a problem we use one or the other, rarely both

Reading a number iteratively, digit by digit

```c
#include <ctype.h>    // for isdigit()
#include <stdio.h>    // for getchar(), ungetc(), stdin
unsigned readnat(void)
{
  unsigned r = 0;     // accumulates result
  int c;              // character read
  while (isdigit(c = getchar())) // while digit
    r = 10*r + c - '0';          // build number
  ungetc(c, stdin);   // put back char != digit
  return r;
}
int main(void)
{
  printf("number read: %u\n", readnat());
}
```

`ungetc(c, stdin)` puts character `c` back to standard input

Character will be read next time, e.g. on using `getchar()`

Basic text processing: reading all text

Reading all input, doing nothing
(body of while is empty, `;` is the *empty statement*)

```
#include <stdio.h>

int main(void) {
  int c;
  while ((c = getchar()) != EOF);
  return 0;
}
```

Reading and printing all input:

```
  int c;
  while ((c = getchar()) != EOF)
    putchar(c);
```

Basic text processing: finding a char

```
int c;
while ((c = getchar()) != EOF)
  if (c == '.') puts("Found period!");
```

Often, we search for more text of some sort after that character. looking for the first word:

```
#include <stdio.h>
int main(void) {
  int c;
  while ((c = getchar()) != EOF)
    if (c == '.') { // found period
      while (isspace(c = getchar())); // skip whitespace
      // print next word made of letters
      while (isalpha(c = getchar())) putchar(c);
  }
  return 0;
}
```

Reading character by character: filters

Function that reads and prints up to a specified character
returns that character or EOF if reached before that char

```
int printto(int stopchar)    // up to what char ?
{
  int c;
  while ((c = getchar()) != EOF && c != stopchar)
    putchar(c);
  return c;
}
```

DON't forget ( ) `(c=getchar())!=EOF` (assign, then compare)

```
int skipto(int stopchar)     // ignore up to stopchar
{
  int c;
  while ((c = getchar()) != EOF && c != stopchar);
  return c;
}
```

`;` after `while` means an empty loop body. *Don't use by mistake!*

ERRORS with characters and loops

NO! ~~`char c = getchar();`~~ YES: `int c = getchar();`

If `char` is `unsigned char`, `c` will never compare equal to `EOF (-1)`

⇒ will never leave a `while (c != EOF)` loop

If `char` is `signed char`, reading byte 255 becomes `-1 (EOF)`

⇒ a valid char (code 255) will be taken as `EOF` (early stop)

NO! ~~`while (!EOF)`~~ `EOF` is a nonzero constant (`-1`)

thus the condition is always false, the loop is never entered!

YES: `while ((c = getchar()) != EOF)` and careful with the `()` !

NO! ~~`while (c = getchar() != EOF)`~~

`!=` has higher precedence, its result (`0` or `1`) is assigned to `c`

NO! ~~`int c = getchar(); if (c < 5) puts("failed exam");`~~

`c` is ASCII code, not value of a one-digit number. Need `c-'0'`

NO! ~~`while ((c = getchar()) != '\n')`~~ may loop forever!

YES: `while ((c = getchar()) != '\n' && c != EOF)` will exit!