Formal Verification

Temporal Logic. Model Checking Basics

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- *Modeling* systems using finite-state machines
- Formal *specification* of sequencing properties: temporal logic
- *Model checking*: verification by traversing the state graph

Q: What kind of systems can we verify ?

A: systems whose behavior is described *precisely* ⇒ *mathematically*

One of the simplest models: *finite state machine*

states and transitions (informally: “circles and arrows”)

Another view: system *state*: set of all quantities that determine the behavior of the system in time

Representation: every state has unique binary encoding (state variable)

Definition of state: depends on *abstraction* level

Example for a processor: instruction set level; internal organization (incl. pipeline); register transfer level; gate-level; transistor level

– *discrete*, *continuous* or *hybrid* systems

– *finite* (⇒ must be discrete) or *infinite* (continuous systems; programs with recursion or dynamic data structures)

Modeling finite state systems

Finite state machines (automata): defined by *states* and *transitions*
ex. program state = variables + prog. counter; transitions = statements
(finite state if finite types, no recursion, no dynamic data)

Our model: a set $V = \{v_1, v_2, \cdots, v_n\}$ of variables over a domain D
– a *state*: an *assignment* $s : V \to D$ of values for each variable in D

– A *state* (assignment) $\Leftrightarrow$ a *formula* true only for that assignment
$\langle v_1 \leftarrow 7, v_2 \leftarrow 4, v_3 \leftarrow 2\rangle$ $\quad$ $(v_1 = 7) \wedge (v_2 = 4) \wedge (v_3 = 2)$
– A *formula* $\Leftrightarrow$ the set of all assignments that make it true
$\Rightarrow$ *sets of states*: representable by logic formulas, e.g., $v_1 \leq 5 \wedge v_2 > 3$
– A *transition* $s \to s'$ has *two* states $\Rightarrow$ a formula over $V \cup V'$
where $V' =$ copy of V (next state variables)
e.g., $(semaphore = red) \wedge (semaphore' = green)$
– *Transition relation*: set of all transitions = a formula $\mathcal{R}(V, V')$

Modeling with Kripke structures

Kripke structure = finite-state automaton with labeled *states*

$$M = (S, S_0, R, L)$$

(compare with automata: labels (input symbols) on *transitions*)

S: finite set of states

$S_0 \subseteq S$: set of initial states

$R \subseteq S \times S$: transition relation

transition relation is *total* if every state has at least one transition

$$\forall s \in S \ \exists s' \in S \ . \ (s, s') \in R$$

$L : S \to \mathcal{P}(AP)$: state labeling function $\mathcal{P}$: powerset (set of subsets)

where AP = set of atomic propositions (observable boolean features that appear in formulas, properties, specifications). Examples:

a state is *stable* (or not)

define the proposition: $bad ::= number\_of\_errors > 0$

Path (trajectory) from a state s_0: *infinite* sequence of states:

$$\pi = s_0 s_1 s_2 \ldots, \text{ such that } R(s_i, s_{i+1}) \text{ for all } i \geq 0$$

Nondeterminism

Transitions are given as a *relation*, not a *function*.
⇒ there can be several states s' such that $s \to s'$, i.e., $(s, s') \in R$
In this case the model (Kripke structure) is called *nondeterministic*
(the future behavior in a state is not uniquely determined).

This is different from the DFA / NFA distinction: finite state automata have *transitions labeled* with *input symbols*
⇒ deterministic if unique next state for given state *and* input symbol
(even if different inputs can lead to different states)

For systems viewed as *open* (interacting with an environment), this is called *input nondeterminism*
Typically, we view Kripke models as *closed*; we will discuss possible parallel composition with an environment

Expressing behavior

Input-output (functional) behavior is not enough for many systems:

Reactive systems interact with environment: *reaction* to a *stimulus*

$\Rightarrow$ Often have infinite execution (operating systems, schedulers, servers)

$\Rightarrow$ A *computation* is an *infinite sequence of states*

Desired properties:

A given (error) state is not reached (*reachability problem*)

The system does not deadlock (*deadlock freedom*), etc.

More general properties can be described in temporal logic

= a *modal* logic, i.e., truth is qualified (possibly, always, etc.)

In this case: with *temporal* modalities: before, after, in the future, ...

– used already by ancient philosophers for reasoning about time

– formalized and applied by Pnueli (1977) to concurrent programs

Linear Temporal Logic (LTL)

Defined by Amir Pnueli in 1977 (ACM Turing Award 1996)

Describes event sequencing along an execution path ⇒ *linear* structure

– an event happens in the future
– a property is invariant (holds everywhere) starting at a given state
– an event follows another event

Temporal operators (truth modalities along an execution trace)

- **X** (*next*): in the next state — also written $\bigcirc$
- **F** (*future*): sometime in the future — $\diamond$
- **G** (*globally*): in every future state (including now) — $\square$

  unary operators, refer to *one* property

- **U** (*until*)

  binary operator, $property_1$ *until* $property_2$

Sometimes also: *release* operator **R** (dual to until). Ignored here.

LTL Syntax

Express that a property is true *for all* paths

$\Rightarrow$ using the *universal quantifier* **A**

$\Rightarrow$ LTL formulas are of of the form $\mathbf{A}\, f$, where f is a *path formula*

Syntax of path formulas:

f ::= p base case: $p \in AP$ is an atomic proposition

$\mid \neg f_1 \mid f_1 \vee f_2 \mid f_1 \wedge f_2$ usual boolean connectors

$\mid \mathbf{X}\, f_1 \mid \mathbf{F}\, f_1 \mid \mathbf{G}\, f_1 \mid f_1\, \mathbf{U}\, f_2$ temporal operators

Since the **A** quantifier is mandatory, and appears only once, it is sometimes left implicit (some authors write path formulas only)

LTL Semantics

LTL formulas of the form $\mathbf{A}\, f$ have their meaning defined in a *state*
$\Rightarrow$ called *state formulas*: true if all paths from s satisfy f

Path formulas have their meaning (truth value) defined over a path.

Notations:

$M, s \models f$ in the model (Kripke structure) M, state s satisfies f

$M, \pi \models f$ in model M, path π satisfies f

If M is fixed (given), we simply write $s \models f$, $\pi \models f$

π^i = suffix of path $\pi = s_0 s_1 s_2 \ldots$ starting at s_i : $s_i s_{i+1} s_{i+2} \ldots$

Semantics of state formulas:

$s \models p \quad \Leftrightarrow \quad p \in L(s)$ (state s has p as a label)

$s \models \mathbf{A}\, f \quad \Leftrightarrow \quad \pi \models f$ for all paths π from s

For path formulas, define semantics as usual by *structural induction*:
the semantics of a formula is given in terms of its simpler subformulas

LTL semantics: path formulas

Semantics of path formulas:

$\pi \models p \quad \Leftrightarrow \quad s \models p$ — $p \in AP$ holds in path origin

$\pi \models \neg f \quad \Leftrightarrow \quad \pi \not\models f$

$\pi \models f_1 \vee f_2 \quad \Leftrightarrow \quad \pi \models f_1 \vee \pi \models f_2$

$\pi \models f_1 \wedge f_2 \quad \Leftrightarrow \quad \pi \models f_1 \wedge \pi \models f_2$

$\pi \models \mathbf{X}\, f \quad \Leftrightarrow \quad \pi^1 \models f$

f holds on the path suffix starting from state 1

$\pi \models \mathbf{F}\, f \quad \Leftrightarrow \quad \exists k \geq 0 \,.\, \pi^k \models f$

there exists a suffix on which f holds (f holds in a state)

$\pi \models \mathbf{G}\, f \quad \Leftrightarrow \quad \forall k \geq 0 \,.\, \pi^k \models f$

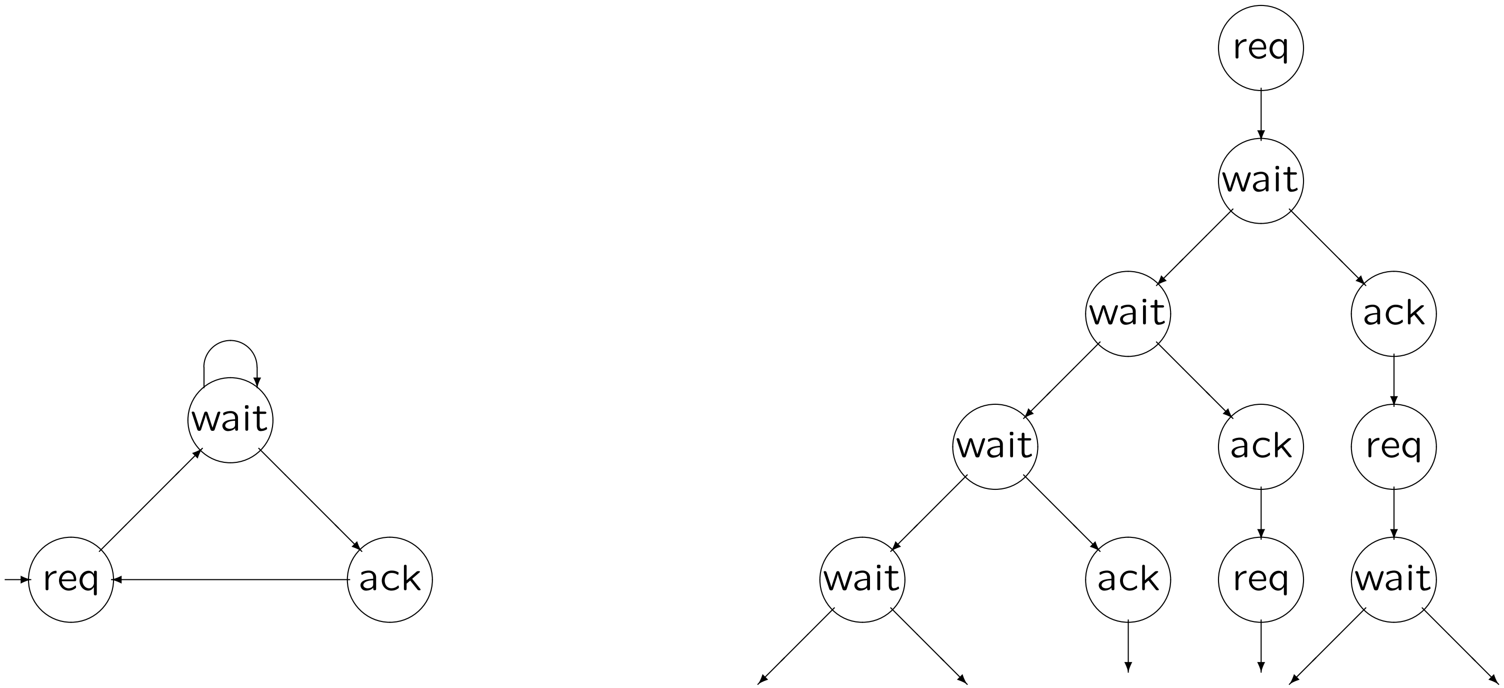
f holds on all path suffixes (f holds in all states)

$\pi \models f_1 \,\mathbf{U}\, f_2 \quad \Leftrightarrow \quad \exists k \geq 0 \,.\, \pi^k \models f_2 \wedge \forall j < k \,.\, \pi^j \models f_1$

f_2 holds on path starting at k (for some k), f_1 holds everywhere prior

The temporal logic CTL*

LTL is a *linear* logic: paths are viewed independently; there may be many futures from origin, but can't express branching *at each step*
⇒ not expressive enough (e.g., always *possible* to reach a state)
⇒ another model: *computation trees* (branching view)
finite unfolding of a state-transition graph starting from an initial state



CTL* syntax and semantics

Additional path quantifier: **E** there exists (a path) $\exists$

Two classes of formulas:

state formulas, evaluated in a state

$f ::= p$ base case: $p \in AP$ atomic proposition

$\quad | \neg f_1 \mid f_1 \vee f_2 \mid f_1 \wedge f_2$ f_1, f_2 state formulas

$\quad | \mathbf{E}\, g \mid \mathbf{A}\, g$ g path formula

path formula, evaluated over a path

$g ::= f$ base case: f is state formula

$\quad | \neg g_1 \mid g_1 \vee g_2 \mid g_1 \wedge g_2$

$\quad | \mathbf{X}\, g_1 \mid \mathbf{F}\, g_1 \mid \mathbf{G}\, g_1 \mid g_1 \,\mathbf{U}\, g_2$

(same rules as LTL, only base case more complex/expressive)

Semantics: same rules as LTL, plus:

$s \models \mathbf{E}\, g \Leftrightarrow$ there exists a path π from s with $\pi \models g$

Computation tree logic CTL

defined by Clarke & Emerson (1981)

$\Rightarrow$ Turing Award 2007 with J.Sifakis for model checking

Tradeoff: expressiveness of specifications vs. efficiency of checking

$\Rightarrow$ CTL is subset of CTL*, efficient to check, enough in many cases

CTL is a *branching-time* logic, like CTL*

CTL *quantifies* over paths starting from a state

$\Rightarrow$ operators **X** , **F** , **G** , **U** are immediately preceded by **A** sau **E**

$\Rightarrow$ syntax of path formulas simplified, directly using state formulas:

$$g ::= \mathbf{X}\, f \mid \mathbf{F}\, f \mid \mathbf{G}\, f \mid f_1\, \mathbf{U}\, f_2 \mid f_1\, \mathbf{R}\, f_2$$

Expressiveness: LTL and CTL incomparable (neither includes the other); both less expressive than CTL*

Relations between operators

$f \wedge g \equiv \neg(\neg f \vee \neg g)$

$\mathbf{F}\, f \equiv \mathit{true}\, \mathbf{U}\, f$

$\mathbf{G}\, f \equiv \neg \mathbf{F} \neg f$

$\mathbf{A}\, f \equiv \neg \mathbf{E} \neg f$

$\Rightarrow$ Operators $\neg$, $\vee$, **X**, **U** and **E** suffice to express any *CTL** formula.

CTL has 2 x 4 = 8 pairs of quantifier $\times$ temporal operator:

$\mathbf{AX}\, f \equiv \neg \mathbf{EX} \neg f$

$\mathbf{EF}\, f \equiv \mathbf{E}\, [\mathit{true}\, \mathbf{U}\, f]$

$\mathbf{AF}\, f \equiv \neg \mathbf{EG} \neg f$

$\mathbf{AG}\, f \equiv \neg \mathbf{EF} \neg f$

$\mathbf{A}\, [f\, \mathbf{U}\, g] \equiv \neg \mathbf{EG} \neg g \wedge \neg \mathbf{E}\, [\neg g\, \mathbf{U}\, (\neg f \wedge \neg g)]$

$\Rightarrow$ all of them expressible using **EX**, **EU** and **EG**

Sample CTL formulas

- $\mathbf{EF}\, \mathit{finish}$
  It is possible to get to a state in which $\mathit{finish} = \mathit{true}$.
- $\mathbf{AG}\, (\mathit{send} \rightarrow \mathbf{AF}\, \mathit{ack})$
  Any *send* is eventually followed by an *ack*.
- $\mathbf{AF}\, \mathbf{AG}\, \mathit{stable}$
  On any path, *stable* is invariant (always holds) after some point
- $\mathbf{AG}\, (\mathit{req} \rightarrow \mathbf{A}\, [\mathit{reg}\, \mathbf{U}\, \mathit{grant}])$
  A *req* stays active until a *grant* is issued.
- $\mathbf{AG}\, \mathbf{AF}\, \mathit{ready}$
  On any path *ready* holds infinitely often.
- $\mathbf{AG}\, \mathbf{EF}\, \mathit{restart}$
  From any state, it is possible to reach a state labeled *restart*.

Model checking. Problem setting

Given a Kripke structure $M = (S, S_0, R, L)$ and a temporal logic formula f, find which states from S satisfy f: $\{s \in S \mid s \models f\}$

Def: A formula (spec.) f holds in M iff all initial states satisfy f:

$$M \models f \stackrel{\text{def}}{=} \forall s_0 \in S_0 \,.\, s_0 \models f$$

History

– independently due to Clarke & Emerson; Queille & Sifakis (1981).

– initially: $10^4 - 10^5$ states. Now: to 10^{100} states (symbolic checking)

Model checking for CTL

By structural decomposition of formula f: compute truth of *all* subformulas of f for each $s \in S$.

– initially, set $l(s) = L(s)$ (atomic propositions true in state s)

– trivial for logical connectors $\neg, \vee, \wedge$

– **EX** f: just label each state that has a successor labeled with f.

– to discuss: two algoritms for basic operators **EU** and **EG**

CTL model checking. The operator EU

Idea: backwards traversal from states labeled f_2 as long as f_1 holds

procedure *CheckEU*(f_1, f_2)

$T := \{s \mid f_2 \in l(s)\}$	if f_2 holds in s
forall $s \in T$ **do** $l(s) := l(s) \cup \{\mathbf{E}\,[f_1\,\mathbf{U}\,f_2]\}$;	then $E[f_1\,\mathbf{U}\,f_2]$ holds, label s
while $T \neq \emptyset$ **do**	still have candidates for search
choose $s \in T$;	
$T := T \setminus \{s\}$;	never consider s twice
forall $s_1 \,.\, R(s_1, s)$ **do**	for all predecessors of s
if $\mathbf{E}\,[f_1\,\mathbf{U}\,f_2] \notin l(s_1) \wedge f_1 \in l(s_1)$ **then**	s_1 not labeled but f_1 holds
$l(s_1) := l(s_1) \cup \{\mathbf{E}\,[f_1\,\mathbf{U}\,f_2]\}$;	$E[f_1\,\mathbf{U}\,f_2]$ also holds, label it
$T := T \cup \{s_1\}$;	s_1 is candidate for continuing search

Terminates since S finite and no labeled state reenters T

CTL model checking. The operator EG

Consider only states satisfying f. Traverse backwards starting from strongly connected components (on cycles where f perpetually holds).

procedure *CheckEG*(f) restrict to states where f holds
  $S' := \{s \mid f \in l(s)\}$;
  $SCC := \{C \mid C$ is nontrivial SCC of $S'\}$; at least one edge
  $T := \cup_{C \in SCC}\{s \mid s \in C\}$; all states in SCCs are on cycles
  forall $s \in T$ **do** $l(s) := l(s) \cup \{\mathbf{EG}\, f\}$; thus get labeled
  while $T \neq \emptyset$ **do** still have candidates for backwards search
    choose $s \in T$;
    $T := T \setminus \{s\}$; continue from s only once
    forall $s_1 \,.\, s_1 \in S' \wedge R(s_1, s)$ **do** for all predecessors of s
      if $\mathbf{EG}\, f \notin l(s_1)$ **then** if s_1 not yet labeled
        $l(s_1) := l(s_1) \cup \{\mathbf{EG}\, f\}$; label s_1
        $T := T \cup \{s_1\}$; s_1 is candidate for continuing search

Terminates; will reach at most every state in S'

Fairness

In practice, we check systems subject to "reasonable" assumptions as:
– a request is not ignored forever (by a scheduler/arbiter)
– communication channels do not continually fail (thus, a message being retransmitted is eventually delivered)
These are properties expressible in CTL*, but not CTL.
⇒ need to extend CTL (semantics) with *fairness constraints*

Intuitively: decision fairness = if a decision (several transitions from a state) is repeated infinitely often, each branch is eventually taken
Reformulate: each destination state of the decision is eventually reached

Formally: A fairness constraint is a *formula* in temporal logic.
A path is *fair* iff the constraint is infinitely often true along the path.
II LTL, we would write: $\mathbf{F}\,\mathbf{G}\, assumption \Rightarrow\ conclusion$
In particular: fairness constraint expressed as *set of states*
⇒ a *fair path* passes infinitely often through the set

Model checking CTL with fairness

Augment the Kripke structure $M = (S, S_0, R, L, F)$, with $F \subseteq \mathcal{P}(S)$
(F = set of subsets of states, $\{P_1, \cdots, P_n\}, P_i \subseteq S$)

$$\text{inf}(\pi) \stackrel{\text{def}}{=} \{s \mid s = s_i \text{ for infinitely many } i\}$$

(set of states appearing infinitely often on π)

π is a *fair* path $\Leftrightarrow \forall P \in F \,.\, \text{inf}(\pi) \cap P \neq \emptyset$.
(π passes infinitely often through each set from F)

For $\models_F$, ("holds fairly") replace "path" with "fair path" in semantics

For model checking, define new atomic proposition *fair*:

$$\textit{fair} \in L(s) \;\Leftrightarrow\; M, s \models_F \textbf{EG}\, \textit{true}$$

$\Rightarrow$ fair-CTL model checking reduces to CTL for $AP \cup \{\mathit{fair}\}$

Complexity of model checking algorithms

– CTL model checking: $O(|f| \cdot (|S| + |R|))$
(linear in size of model and formula)
– CTL with fairness: $O(|f| \cdot (|S| + |R|) \cdot |F|)$
– LTL: PSPACE-complete $|M| \cdot 2^{O(|f|)}$
(different type of algorithm, based on a *tableau construction*)
– CTL*: like LTL $|M| \cdot 2^{O(|f|)}$

CTL: usually preferred, because of polynomial (linear!) algorithm
Spin uses LTL: exponential only in size of formula (usually small)

Synchronous and asynchronous composition

Behavior of composed systems emerges from component behavior.

For concurrently executing components: *parallel* composition:

- synchronous: conjunction (simultaneous transitions)

$$R(V, V') = R_1(V_1, V_1') \wedge R_2(V_2, V_2') \qquad V = V_1 \cup V_2$$

- asynchronous: disjunction (individual transitions)

$$R(V, V') = R_1(V_1, V_1') \wedge \mathit{Eq}(V \setminus V_1) \vee R_2(V_2, V_2') \wedge \mathit{Eq}(V \setminus V_2)$$

$$\mathit{Eq}(U) = \bigwedge_{v \in U} (v = v')$$

  – arbitrary interleaving between transitions of components
  – a transition modifies just the variables of *one* component
  – simultaneous transitions are deemed impossible