## Symmetric Primitives

(block ciphers, stream ciphers, hash functions, keyed hash functions and (pseudo)random number generators)

An informal, yet instructive account of symmetric primitives ...

## Begin with an informal question

- Question: What do you expect from cryptography?
- (Potentially correct) Answer: Protect you stored data \& ongoing communications (let's call this simply protect messages)
- Question: Assume you are given am encryption box (call it symmetric encryption) that encrypts your data with a key. Is your data now protected?
- (At least incomplete) Answer: Yes, as long as the adversary cannot find/guess the


Lorem ipsum dolor sit amet
 key ... or maybe not

## A practical example - the Enigma machine

- A rotor cipher machine (several versions of it), elements:
- 26 lamps (output, ciphertext) \& keys (input, plaintext)
- 3 or 5 (usually) rotors
- at most 13 plugs that can connect each two letters on the plug-board (part of the key)

a) rotors (3)
b) lamps (26)
c) keys (26)
d) plugboard ( $2 \times 13$ )



## How Enigma works

- When one key is pressed (letter of the plaintext selected), circuit is closed under that key, current flows through the plugboard that follows the 3 rotors, returns from the reflector and lightens up the lamp (the letter of the ciphertext)
- Rotors move at each step, thus a character will not always get encrypted to the same character (i.e., a polyalphabetic substitution)
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- A closer look to the Enigma secret key (depicted in the form of a codebook) may give you more insights on the security of this cryptosystem



## How secure is Enigma

- Question: how hard is to break Enigma?
- Answer (not necessarily correct): as hard as to find the key
- Question: how big is Enigma's key?
- Answer: consider just (the way to place 3 rotors) x (the way to connect 13 plugs)

$$
26^{3} \times \frac{26!}{13!\times 2^{13}}=138953282533065000
$$

when compared to the number of DES keys $2^{56}=72057594037927936$ will quickly lead to the conclusion that Enigma (deprecated by the end of WW2) is stronger than DES (deprecated only by the end of the ' 90 s)

## How secure is Enigma

- Question: imagine you have captured a ciphertext that begins with:

> zeyt sadb dikf dsak sadk jnujj

Could you tell which is the corresponding plaintext from the following:
a) attackatdawnonthewestfront
b) attackatnightonthewestfront
c) attackatduskonthewestfront

- Answer: wrong design decision in Enigma, a letter cannot map to itself! Correct answer is c)


## Partial conclusion

- For protecting data by symmetric primitives we need: clear design principles (how to build the ciphers) and a formal treatment of security properties (what is the exact security they should offer)

A more formal and constructive account of symmetric primitives ...
you should learn:
i. where is the primitive used,
ii. what are the standards,
iii. how is it built,
iv. what are its properties

## Type of functions (I) Symmetric encryption schemes

- Description (informal): an algorithm that takes as input a key $k$ and message $m$ called plaintext and returns the encrypted message c called ciphertext (similarly, algorithms for decryption and generating keys are needed)

- Example of use: encrypted tunnels SSL/TLS, IPSEC; encrypted passwords (Imhash in Win XP); encrypted hard drives (TrueCrypt), etc.
- Standards:
$>$ Not to use: DES, RC4
$>$ To use: AES (128, 194, 256), 3DES (with 168 bit key, not recommended)


## Symmetric encryption: formal definition

- A symmetric encryption scheme is a triple of algorithms:
$>$ Gen is the key generation algorithm that takes random coins, a security parameter ( $l$ ) and outputs the key

$$
k \leftarrow \operatorname{Gen}\left(1^{l}\right)
$$

$>$ Enc is the encryption algorithm that takes as input the key and some message, then outputs the ciphertext

$$
c \leftarrow \operatorname{Enc}(k, m)
$$

$>$ Dec is the decryption algorithm that takes as input the ciphertext and the key and outputs the message

$$
m \leftarrow \operatorname{Dec}(k, c)
$$

- A correctness condition enforces that $\operatorname{Dec}(k, \operatorname{Enc}(k, m))=m$
- In some cases, the encryption and decryption algorithms are allowed to return \null on particular inputs (i.e., they refuse to encrypt/decrypt)


## Design principle: product ciphers

- Substitutions and transpositions (suggested in the work of Shannon, also used before)
>Substitution (S-Box) replaces a symbol (or group of symbols) by another symbol - creates confusion
>Permutations (P-Box) also known as transpositions mixes the symbols inside a block creates diffusion
- Ciphers that use both substitutions and permutations (S-Boxes and P-boxes) are also called product ciphers (sometimes product ciphers denote any cipher that uses more than one transformation, while product ciphers with only S\&P are called SP-networks)
- Remarks:
$>$ DES and AES, the two well known standards are product ciphers
$>$ Feistel ciphers are also product ciphers


## Classification: block ciphers vs. stream ciphers

- Stream ciphers - the message is combined via a simple transformation (e.g. XOR) with a keystream (which is a pseudorandom stream generated by a more complex mechanism), operation is done one character (bit) at a time. Examples include RC4 used in SSL/TLS or A5 used in GSM.
- Block ciphers - the message is transformed block by block (e.g., 128 bits) via a transformation that is depended on the key. Examples include DES, 3DES, AES.
- Remarks:
- Block ciphers can be turned into stream ciphers in certain mode of operations, e.g., counter mode (this means that distinction between the two is not always clear)
- Typically stream ciphers have low hardware complexity, are fast, but practical instantiations such as RC4 are not always secure



## Example: the one-time pad (a stream cipher)

- Question: could you build a cipher that cannot be broken regardless of the computational power of the adversary?
- Answer: believe it or not, yes. The one-time pad is information-theoretically secure, i.e., cannot be broken regardless of computational power \& ciphertext available.
- Description: generate a random key the same length as the plaintext, then simply XOR it with the plaintext

- Problems:
- requires a random key stream the same length as the plaintext, but in practice you want a key as small as possible
- Since it's symmetric the key needs to be exchanged a-priori on a secure channel, but then why not simply exchange the plaintext?
- Current status: there are still some practical applications where it's useful, e.g., quantum cryptography, otherwise it is not an efficient solution


## Design: Feistel networks

- Designed by Horst Feistel in the '70s at IBM
- SP-networks
- How they work:
$>$ Variable number of rounds
$>$ Each block is split into right and left part (if equal in size, then the network is called balanced)
$>$ Right block is passed through a round function that depends on the round key
$>$ Round key is derived from the master key (via the key scheduling algorithm)
$>$ Security/performance trade-off: increasing the number of rounds and the size of the key results in increasing security level
$>$ Decryption is performed by walking through



## Relevant property of the Feistel round

- Note that the Feistel round is invertible regardless of the properties of the round function, so inverting the network is straight forward as follows
- By definition, deriving the output from the input:

$$
L_{i}=R_{i-1}, R_{i}=L_{i-1} \oplus f_{i}\left(R_{i-1}\right)
$$

- Which implies, deriving the input from the output

$$
R_{i-1}=L_{i}, L_{i-1}=R_{i} \oplus f_{i}\left(L_{i}\right)
$$

## Design insights: DES



## DES round function

- How it works: the right half (32 bit) of the message block ( 64 bit ) is expanded ( 48 bit ) then XOR-ed with the round key ( 48 bit ) and each 6 bits are provided as input to $8 \times \mathrm{S}$ Boxes that output only 4 bits resulting in 32 bits that are passed through another permutation $P$
- This round transformation is applied 16 times, each time with a distinct round key



## Examples: E, P and some S-boxes (from the standard)

$$
\begin{aligned}
& E=\left(\begin{array}{llllll}
32 & 1 & 2 & 3 & 4 & 5 \\
4 & 5 & 6 & 7 & 8 & 9 \\
8 & 9 & 10 & 11 & 12 & 13 \\
12 & 13 & 14 & 15 & 16 & 17 \\
16 & 17 & 18 & 19 & 20 & 21 \\
20 & 21 & 22 & 23 & 24 & 25 \\
24 & 25 & 26 & 27 & 28 & 29 \\
28 & 29 & 30 & 31 & 32 & 1
\end{array}\right) \\
& S_{1}=\left(\begin{array}{llllllllllllllll}
14 & 4 & 13 & 1 & 2 & 15 & 11 & 8 & 3 & 10 & 6 & 12 & 5 & 9 & 0 & 7 \\
0 & 15 & 7 & 4 & 14 & 2 & 13 & 1 & 10 & 6 & 12 & 11 & 9 & 5 & 3 & 8 \\
4 & 1 & 14 & 8 & 13 & 6 & 2 & 11 & 15 & 12 & 9 & 7 & 3 & 10 & 5 & 0 \\
15 & 12 & 8 & 2 & 4 & 9 & 1 & 7 & 5 & 11 & 3 & 14 & 10 & 0 & 6 & 13
\end{array}\right) \\
& S_{2}=\left(\begin{array}{llllllllllllllll}
15 & 1 & 8 & 14 & 6 & 11 & 3 & 4 & 9 & 7 & 2 & 13 & 12 & 0 & 5 & 10 \\
3 & 13 & 4 & 7 & 15 & 2 & 8 & 14 & 12 & 0 & 1 & 10 & 6 & 9 & 11 & 5 \\
0 & 14 & 7 & 11 & 10 & 4 & 13 & 1 & 5 & 8 & 12 & 6 & 9 & 3 & 2 & 15 \\
13 & 8 & 10 & 1 & 3 & 15 & 4 & 2 & 11 & 6 & 7 & 12 & 0 & 5 & 14 & 9
\end{array}\right) \\
& 1152326 \\
& P=\left|\begin{array}{ccccc}
5 & 18 & 31 & 10 \\
2 & 8 & 24 & 14
\end{array}\right| \\
& 3227 \quad 3 \quad 9 \\
& 1913306 \\
& 2211425
\end{aligned}
$$

## DES key scheduling

- Derives each of the round keys from the master key


PERMUTATION 2

## Designs: 3DES

- 3 DES keys K1, K2, K3 in the following transformation:

$$
c=E_{K 3}\left(D_{K 2}\left(E_{K 1}(m)\right)\right), \quad m=D_{K 1}\left(E_{K 2}\left(D_{K 3}(c)\right)\right)
$$

- Considered to be secure so far (given that all three keys are random and independent) but it is slower than AES (thus no serious reasons for use in practice)
- Has 3 keying options: i. independent keys, ii. K1 and K2 independent but K3=K1, iii. all keys are equal K1=K2=K3 (this is DES)
- Main reason for practical persistence may be the electronic payment industry


## Designs: AES

## - AES facts:

$>$ Designed by Vincent Rijmen and Joan Daemen
$>$ Selected by public competition from the 5 finalists: MARS, RC6, Rijndael, Serpent, and Twofish
$>$ The new standard as of 2001
$>$ Not a Feistel network
$>$ Available with 3 key lengths: 128, 192, 256 bits

## - How AES works

$>$ Operates on a $4 \times 4$ matrix of bytes ( 128 bit blocks) called state
$>$ Has 10, 12 or 14 rounds according to the key size
$>$ Each round has 4 transformations: SubBytes (a substitution) is non-linear substitution where each byte is replaced via a look-up table, ShiftRows (a permutation) the last three rows are shifted, MixColumns the four bytes of each column are combined via a linear transformation, AddRoundKey each byte of the state is combined with the round key via a XOR operation

```
AES_Encrypt_Round(State, Key)
{
    SubBytes(State);
    ShiftRows(State);
    MixColumns(State);
    AddRoundKey(State, Key);
}
```

```
AES_Decrypt_Round(State, Key)
```

AES_Decrypt_Round(State, Key)
{
{
AddRoundKey}\mp@subsup{}{}{-1}\mathrm{ (State, Key);
AddRoundKey}\mp@subsup{}{}{-1}\mathrm{ (State, Key);
MixColumns }\mp@subsup{}{}{-1}\mathrm{ (State);
MixColumns }\mp@subsup{}{}{-1}\mathrm{ (State);
ShiftRows }\mp@subsup{}{}{-1}\mathrm{ (State);
ShiftRows }\mp@subsup{}{}{-1}\mathrm{ (State);
SubBytes }\mp@subsup{}{}{-1}\mathrm{ (State);
SubBytes }\mp@subsup{}{}{-1}\mathrm{ (State);
}

```
}
```


## Block Ciphers use in practice

- Question: block ciphers work on single blocks of message, how do you extend them to multiple blocks?


## Electronic Code Book (ECB)

- The message is parsed into blocks and each block is encrypted with the secret key
- Decryption is done by reversing this operation

- Question: assuming that the block cipher is secure, is this construction secure?
- Answer: No. Do not use ECB.
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Original


Encrypted using ECB mode

## Cipher Block Chaining (CBC)

- Initialization Vector (IV) is a non-secret random value used for randomization of the first output block
- Last message chunk is padded to the block length
- Pros: encryption is fully randomized and secure

- Cons: if one of the blocks is lost, decryption cannot be performed

Some variations: Output-feedback (OFB) and Cipher Feedback (CFB)

- Pros: OFB allows decryption even when message blocks are lost, it also allows precomputation of the key stream



## Another variation: Propagating Cipher Block Chaining (PCBC)



## Counter Mode

- A counter is incremented and encrypted for each block, then XORed with the message
- Pros: decryption can still be performed if bocks are lost, key-stream can be precomputed
- This mainly converts the block cipher into a stream cipher



## Adversary capabilities (informal) - what the adversary can do?

- CPA - chosen plaintext adversary, an adversary that has access to a black-box that encrypts plaintexts at the adversary choice
- CCA - chosen ciphertext adversary, an adversary that has access to a black-box that decrypts cyphertexts at the adversary choice
- Adaptive vs. non-adaptive - is an additional flavour that can be added to both CPA and CCA meaning that the adversary can continue (adaptive) or not (non-adaptive) to query the encryption/decryption box after he received the target ciphertext that he is required to break (obviously the adversary is not allowed to query the target ciphertext to the decryption box)


## Security notions (informal)

- semantic security (SS) (Goldwasser \& Micali 1982)

Any information that can be efficiently computed with the ciphertext, can be also computed without the ciphertext

## - indistinguishability of ciphertexts (IND)

Given two messages selected by the adversary and the encryption of one of them chosen at random (without adversary's knowledge) the adversary cannot decide which is the encrypted message

## - real or random indistinguishability (RoR)

Given a message selected by the adversary and the encryption of either this message or some complete random message (not known to the adversary) the adversary cannot decide if the ciphertext corresponds or not to its chosen plaintext

## - Question: which of the previous properties is the strongest?

- Answer: under proper formalization they are all equivalent, see Goldreich Foundations of Cryptography, vol II, p. 383
- Question: which is easier to prove?
- Answer: generally IND or RoR are easier to prove and are the standard tool in proving security


## How to prove equivalences?

- Security reductions, proving that a cryptosystem that has one property has the other (or the reverse, if it doesn't have one property it doesn't have the other)


## Example, security reductions: IND $\rightarrow$ RoR \& IND $\leftarrow$ RoR

- Proof to be done as exercise during laboratory hours


## Type of functions (II) Hash functions

- Description: an algorithm that takes as input a message of any length and turns it into a constant size output (usually referred as tag or simply hash)

- Example of use: assure integrity of software downloads/updates, protect stored passwords, etc.
e.g., downloading images from ubuntu.com


### 14.10

(Utopic Unicorn): October 2014 (Supported until July 2015)

- Standards:

4a3c4b8421af51c29c84fb6f4b3fe109 ubuntu-14.10-desktop-i386.iso
$>$ Not to use MD5, SHA1 (not resistant to collisions)
91bd1cfba65417bfa04567e4f64b5c55 ubuntu-14.10-server-amd64.iso
$>$ To use SHA2 (mostly 256, 384 and 512 are somewhat slow)
$\Rightarrow$ Future use: SHA3 (Keccak the winner of the competition)
$>$ Alternatives: BLAKE is a lightweight design, one of the SHA3 finalists

## Security properties for hash functions

- The following properties are mandatory for hash functions:
- Pre-image resistance - given the hash of some message it is infeasible to find the message

$$
\text { i.e., } h(m) \xrightarrow{?} m
$$

- Secondary pre-image resistance - given the hash of a message and the message it is infeasible to find a second message that has the same hash value

$$
\text { i.e., } m_{1}, h\left(m_{1}\right) \xrightarrow{?} m_{2} \text { s.t. } h\left(m_{1}\right)=h\left(m_{2}\right)
$$

- Collision resistance - it is infeasible to find two messages that have the same hash

$$
\text { i.e., } \xrightarrow{?} m_{1}, m_{2} \text { s.t. } h\left(m_{1}\right)=h\left(m_{2}\right)
$$

## Design principle

- The Merkle-Damgard construction provides a method for turning a collisionresistant one-way functions into a collision-resistant hash functions
- This design stands behind MD5, SHA1 and SHA2
- The IV is fixed (not random like in block ciphers modes of operation)

Message block 1


## Design insights: MD5

- 4 IV's defined as follows

$$
\begin{aligned}
& \mathrm{A}=0 \times 67452301 \\
& \mathrm{~B}=0 \times \mathrm{xefcdab} 89 \\
& \mathrm{C}=0 \times 98 \text { badcfe } \\
& \mathrm{D}=0 \times 10325476 .
\end{aligned}
$$

- Message is processed in blocks of 512 bits that are further split in 128 bit chuncks and propagated as IVs for the next block to be hashed (i.e., Merkle-Damgard construction)



## MD5 round function

- Each round proceeds with the following transformation (A, B, C, and D are the IV's, $K$ and $S$ are fixed constants and $M$ is the message):

$$
\begin{aligned}
& D \leftarrow C \\
& C \leftarrow B \\
& B \leftarrow B+((A+F R(B, C, D)+M+K) \lll S) \\
& A \leftarrow D
\end{aligned}
$$

- Round function is distinct for each round (still, all round functions consist in simple logic operations AND, OR, XOR and NOT):

$$
\begin{aligned}
& F(X, Y, Z)=(X \wedge Y) \vee(\neg X \wedge Z), \\
& G(X, Y, Z)=(X \wedge Z) \vee(Y \wedge \neg Z), \\
& H(X, Y, Z)=X \oplus Y \oplus Z, \\
& I(X, Y, Z)=Y \oplus(X \vee \neg Z) .
\end{aligned}
$$

## Test vectors as per RFC 1321

- Examples of what you get after you hash

```
MD5 ("") = d41d8cd98f00b204e9800998ecf8427e
MD5 ("a") = 0cc175.b9c0f1b6a831c399e269772661
MD5 ("abc") = 900150983cd24fb0d6963f7d28e17f72
MD5 ("message digest") = f96b697d7cb7938d525a2f31aaf161d0
MD5 ("abcdefghijklmnopqrstuvwxyz")=c3fcd3d76192e4007dfb496cca67e13.b
MD5 ("ABCDEFGHIJKLMNOPQRSTUVWXYZabcdefghijklmnopqrstuvwxyz0123456789")
    = d174ab98d277d9f5a5611c2c9f419d9f
MD5 ("12345678901234567890123456789012345678901234567890123456...2345678
90") = 57edf4a22be3c955ac49da2e2107b67a
```


## Type of functions (I) Keyed Hash Functions (or MACs)

- Description (informal): an algorithm that takes a message of arbitrary length and and a key then outputs a tag

- Example of use: assuring message authentication, i.e., binding a message with the identity of a principal that knows a key
- Standards:
$>$ Not to use: simple concatenation of key to a message is in general insecure
$\rightarrow$ To use: HMAC or NMAC with one of the previous hash functions
PFuture use: N/A


## Message Authentication Codes formal definition

- A message authentication code is a triple of algorithms:
$>$ Gen is the key generation algorithm that takes random coins, a security parameter l and outputs the key

$$
k \leftarrow \operatorname{Gen}\left(1^{l}\right)
$$

$>$ Mac is the tag-generation algorithm that takes as input the key and some message, then outputs the tag

$$
t a g \leftarrow M A C(k, m)
$$

$>$ Ver is the verification algorithm that takes as input the key, the tag and the message and outputs 1 if the tag is valid or 0 otherwise

$$
m \leftarrow \operatorname{Ver}(k, t a g, m)
$$

- A correctness condition enforces that $\operatorname{Ver}(k, \operatorname{MAC}(k, m), m)=1$


## Desired Properties for MACs

- Fortunately, there is only one strong definition of security (of course, this can be refined in several ways)
- MACs must have (existential) unforgeability under chosen message attacks, that is, an adversary that receives any number of valid message-tag pairs (i.e., pairs that are computed with the MAC algorithm) is unable to output a new messagetag pair that will successfully pass through the verification algorithm


## What not to use

- Question: based on the previous security definition for MAC code, is the simple concatenation of message to key, i.e., $\mathrm{H}(\mathrm{k}|\mid \mathrm{m})$, secure?
- Answer: No. Concatenation attacks are possible due to the construction of some hash functions (revisit MD5 and the Merkle-Damgard construction)

Message block 1


## HMAC

- Simple and secure
- The application of a hash function twice with an inner-padding (ipad) and outerpadding (opad)
- ipad is B blocks of $0 \times 36$ and opad is B blocks of $0 \times 5 C$, where $B$ is the byte size of the block to be processed (e.g., $B=64$ in case of MD5 that uses blocks of 512bits)

$$
H M A C(K, m)=H((K \oplus \mathrm{opad}) \| H((K \oplus \mathrm{ipad}) \| m))
$$

- Can be paired with any hash function, e.g., HMAC-MD5, HMAC-SHA256, etc.
- NMAC (Nested MAC) is as simple as HMAC, however it requires changing the IV which is less handy when implementing


## Various paradigms of combining MACs with encryptions

- A frequent application of MAC functions is in authenticated encryption, i.e., assuring that an encrypted ciphertext indeed originates from the source (note that block ciphers are not designed for this)
- There are three paradigms employed in practice:
- Encrypt-and-MAC, i.e., $E_{k}(m) \| M A C_{k}(m)$, used in SSH
- MAC-then-encrypt , i.e., $E_{k}\left(m \| M A C_{k}(m)\right)$, used in SSL/TLS
- Encrypt-then-MAC, i.e., $E_{k}(m) \| M A C_{k}\left(E_{k}(m)\right)$, used in IPSec
- Encrypt-then-MAC has better security than the previous two and should be the desired alternative in practice
- For details, see Bellare \& Namprempre, "Authenticated Encryption: Relations among notions and analysis of the generic composition paradigm", 2000


## Type of functions (IV) RNGs and PRNGs

- Random numbers stay at the core of any cryptosystem since you need randomness for the secret keys
- Description (informal):
- TRNG - True random-number generators output random sequences based on physical processes that are hard/infeasible to model, i.e., white noise from a Zenner diode, oscillator drift, SRAM state at power-up, etc.
- PRNGs - deterministic algorithms that generate a random sequence based on a value called seed (they all have cycles but this does not mean they are insecure, computationally secure PRNGs exist)
- Example of use: used in any handshake SSL/TLS, IPSec, etc. that needs to generate a fresh session key


## PRNG examples

- The linear congruential generator, an insecure and yet common solution

$$
X_{i+1}=a X_{i}+c \bmod n \quad\left(X_{0} \text { is the seed }\right)
$$

- Galois or Fibonacci LFSR (Linear Feedback Shift Register) are another common, insecure alternative

- Bloom-Bloom-Shub is cryptographically secure but requires a large modulus $n$ and is computationally expensive, thus almost absent in practice ( $X_{0}$ is the seed)

$$
X_{i}=X_{i-1}^{2} \bmod n \quad\left(X_{0} \text { is the seed }\right)
$$

- Block ciphers in counter mode or stream ciphers provide secure instantation of PRNGs (as long as the cipher is secure)


## How to test RNG \& PRNGs

- Various statistical tests are usually employed, none is perfect but may provide some degree of confidence
- Dieharder is a battery of tests used by many enthusiasts or professionals http://www.phy.duke.edu/~rgb/General/rand_rate.php


## Questions?

