Karnaugh Maps: Problems

For problems 1, 2 and 3, for each function (if possible !), implement with gates in VHDL the initial function and the minimized function, and compare them by simulation.

1. Using Karnaugh maps, find a minimal sum-of-products expression for each of the following logic functions. Indicate the distinguished 1-cells in each map.

   (a) $F = \sum_{X,Y,Z}(1, 3, 5, 6, 7)$
   (b) $F = \sum_{W,X,Y,Z}(1, 4, 5, 6, 7, 9, 14, 15)$
   (c) $F = \prod_{W,X,Y}(1, 4, 5, 6, 7)$
   (d) $F = \sum_{W,X,Y,Z}(0, 1, 6, 7, 8, 9, 14, 15)$
   (e) $F = \prod_{A,B,C,D}(4, 5, 6, 13, 15)$
   (f) $F = \sum_{A,B,C,D}(4, 5, 6, 11, 13, 14, 15)$

2. Using Karnaugh maps, find a minimal sum-of-products expression for each of the following logic functions. Indicate the distinguished 1-cells in each map. Find the complete sum for the logic functions from (d) and (e).

   (a) $F = \sum_{A,B,C}(0, 1, 2, 4)$
   (b) $F = \sum_{W,X,Y,Z}(1, 4, 5, 6, 11, 12, 13, 14)$
   (c) $F = \prod_{A,B,C}(1, 2, 6, 7)$
   (d) $F = \sum_{W,X,Y,Z}(0, 1, 2, 3, 7, 8, 10, 11, 15)$
   (e) $F = \sum_{W,X,Y,Z}(1, 2, 4, 7, 8, 11, 13, 14)$
   (f) $F = \prod_{A,B,C,D}(1, 3, 4, 5, 6, 7, 9, 12, 13, 14)$

3. Re-do the prime-number-detector minimization, assuming that 1 is not a prime number.

Problems with don't care input combinations.

4. Using Karnaugh maps, find a minimal sum-of-products expression for each of the following logic functions. Indicate the distinguished 1-cells in each map. Find all prime implicants and the essential prime implicants.

   (a) $G = \sum_{W,X,Y,Z}(0, 1, 3, 5, 14) + d(8, 15)$

   (b) $G = \sum_{W,X,Y,Z}(0, 1, 2, 8, 11) + d(3, 9, 15)$

   (c) $G = \sum_{A,B,C,D}(4, 6, 7, 9, 13) + d(12)$

   (d) $G = \sum_{A,B,C,D}(1, 5, 12, 13, 14, 15) + d(7, 9)$

   (e) $G = \sum_{W,X,Y,Z}(4, 5, 9, 13, 15) + d(0, 1, 7, 11, 12)$

5. Re-do the minimization for the prime-number-detector for BCD (binary codded decimally) numbers, considering that 1 is not a prime number.

   Problems with product of sums:

6. Using Karnaugh maps, find a minimal product of sums expression for each of the logic functions from exercises 1 and 2.