Examples of clocked synchronous state machines

Doru Todinca

Department of Computers and Information Technology
Politehnica University of Timisoara

Outline

Outline

Example 1: a modulo 6 binary count-up counter

Example 2: a modulo 6 binary count-down counter

Example 3: a zero's counter

Example 1: a modulo 6 binary count-up counter

Design a clocked synchronous state machine which counts cyclically in binary ascending order modulo 6. Obtain the excitation equations in the following cases:

(a) *for an implementation with D flip-flops, minimal cost approach*
(b) *for an implementation with D flip-flops, minimal risk approach*
(c) *for an implementation with T flip-flops with enable, minimal cost approach*
(d) *for an implementation with T flip-flops with enable, minimal risk approach*

Solution

- Since it is a modulo 6 counter, it means that we have 6 states, the decimal numbers from 0 to 5.
- Because we have 6 states, we need 3 state variables.
- For a binary counter, the state encoding is not necessary, since each state represents a binary encoding of the corresponding decimal number.
- In this case, there are the decimal numbers from 0 to 5, which means, the binary numbers from 000 to 101.
- The counting sequence is $0 \to 1 \to 2 \to 3 \to 4 \to 5 \to 0$
- Since the state coding is not necessary in this case, we can go directly to the transition table (table 1). Here we do not have an output, the states are also the outputs.
- When we use D flip-flops for implementation, since for a D flip-flop D=Q*, the transition table is also the excitation table.

Nr.	**Q2**	**Q1**	**Q0**	**Q2***	**Q1***	**Q0***
0	0	0	0	0	0	1
1	0	0	1	0	1	0
2	0	1	0	0	1	1
3	0	1	1	1	0	0
4	1	0	0	1	0	1
5	1	0	1	0	0	0
				D2	D1	D0

Table 1: Transition table for the modulo 6 counter.

Minimal cost and minimal risk approach

- In this case we have 3 state variables but only 6 states
- The codes 110 and 111, corresponding to the states 6 and 7 are not used.
- The question is, what do we put in the excitation maps (i.e., Karnaugh maps), in the cells that correspond to these 2 inputs
- The easiest way is to put d (don't care), since we know that these states (6 and 7) cannot be reached in the normal functioning of the state machine, so we don't care if the next state of these states will be 0 or 1.
- This is the minimal cost approach, and it is used in most cases

Minimal cost and minimal risk approach

- However, if the state machine that we design is used in safety-critical applications (e.g., if it is part of a device that opens the door of a passenger airplane, or is part of a nuclear plant, etc) we take another approach:
- We know that in normal condition the state machine will not reach the states 6 and 7, but, if due to some malfunctioning, or human error, etc, the machine WILL REACH one of these state, we desing the machine such that the next state is a safe state (e.g., a state where nothing wrong happens).
- Such a state is very often (but not always) the initial state, in our case state 0, encoded 000
- This is called minimal risk approach
- So, in the transition and excitation table, the next state corresponding to states 6 and 7, will be *ddd* for minimal cost approach, and 000 for minimal risk approach (see table 2).

Excitation table for implementation with D flip-flops

Nr.	**Q2**	**Q1**	**Q0**	**Q2***	**Q1***	**Q0***
0	0	0	0	0	0	1
1	0	0	1	0	1	0
2	0	1	0	0	1	1
3	0	1	1	1	0	0
4	1	0	0	1	0	1
5	1	0	1	0	0	0
6	1	1	0	d/0	d/0	d/0
7	1	1	1	d/0	d/0	d/0
				D2	D1	D0

Table 2: Transition and excitation table for the modulo 6 counter, including the states 6 and 7

Cell numbering in a Karnaugh map

		Q2Q1			
		00	**01**	**11**	**10**
Q0	**0**	0	2	6	4
	1	1	3	7	5

Table 3: Cell numbering in a Karnaugh map

Excitation maps

		Q2Q1			
		00	**01**	**11**	**10**
Q0	**0**	0	0	d	1
	1	0	1	d	0

Table 4: Excitation map for D2, minimal cost. We copy the values from table 2, from the column of D2, into the Karnaugh maps.

$\implies D2 = Q2 \cdot Q0' + Q1 \cdot Q0$ (excitation equation for D2, minimal cost approach)

		Q2Q1			
		00	**01**	**11**	**10**
Q0	**0**	0	0	0	1
	1	0	1	0	0

Table 5: Excitation map for D2, minimal risk

$D2 = Q2 \cdot Q1' \cdot Q0' + Q2' \cdot Q1 \cdot Q0$ (excitation equation for D2, minimal risk approach)

Excitation maps and excitation equations for D1

- We can treat together the minimal cost and minimal risk approach, if we want to save time.

		Q2Q1			
		00	**01**	**11**	**10**
Q0	**0**		1	d/0	
	1	1		d/0	

Table 6: Excitation map for D1, minimal cost and minimal risk

$D1 = Q2' \cdot Q1' \cdot Q0 + Q1 \cdot Q0'$ minimal cost
$D1 = Q2' \cdot Q1' \cdot Q0 + Q2' \cdot Q1 \cdot Q0'$ minimal risk

Excitation maps and excitation equations for D0

		Q2Q1			
		00	**01**	**11**	**10**
Q0	**0**	1	1	d/0	1
	1			d/0	

Table 7: Excitation map for D0, minimal cost and minimal risk

$D0 = Q0'$ minimal cost
$D0 = Q2' \cdot Q0' + Q1' \cdot Q0'$ minimal risk

Implementation with T flip-flops with enable

- Obtaining the excitation maps and excitation equations for T flip-flops with enable is a bit more laborious than for D flip-flops
- A F flip-flop with enable remains in the same state if the enable input (we call it EN) has the value 0, and changes its state (from 0 to 1 or from 1 to 0) if $EN = 1$
- Hence, if we want the T flip-flop to change its state, we must have $EN = 1$, and if we want it to maintain its state, we must have $EN = 0$
- When we build the excitation table for T flip-flops with enable, we add in the transition and output table a new column for each pair of present value and next value of each state variable
- So, we will add a column for EN2, which depends on Q2 and Q2*, a column for EN1, which depends on Q1 and Q1*, and a column for EN0, corresponding to Q0 and Q0*.

Implementation with T flip-flops with enable

- As an example, lets look in the line corresponding to the state 2 (i.e. 010), in the table 8
- The next state of state 2 is state 3, encoded 011
- This means that $Q2 = 0$ and $Q2* = 0$, so it follows that $EN2 = 0$ (since Q2 does not change its value)
- $Q1 = 1$ and $Q1* = 1$, which means that Q1 does not change its value, so we will have $EN1 = 0$
- $Q0 = 0$ and $Q0* = 1$, so Q0 changes its value; it results that $EN0 = 1$
- In the line corresponding to state 3, encoded 011, next state is 4, encoded 100, which means that all 3 state variable change their values, so we will have $EN2 = EN1 = EN0 = 1$

Excitation table for implementation with T flip-flops with enable

Nr.	**Q2**	**Q1**	**Q0**	**Q2***	**Q1***	**Q0***	**EN2**	**EN1**	**EN0**
0	0	0	0	0	0	1	0	0	1
1	0	0	1	0	1	0	0	1	1
2	0	1	0	0	1	1	0	0	1
3	0	1	1	1	0	0	1	1	1
4	1	0	0	1	0	1	0	0	1
5	1	0	1	0	0	0	1	0	1
6	1	1	0	d/0	d/0	d/0	d/1	d/1	d/0
7	1	1	1	d/0	d/0	d/0	d/1	d/1	d/1

Table 8: Transition and excitation table for the modulo 6 counter, for T flip-flops

Excitation maps and excitation equations for implementation with T flip-flops

- The procedure for obtaining the excitation maps is the same: we copy the column corresponding to $EN2$ into a Karnaugh map, the column corresponding to $EN1$ into another Karnaugh map, and the column corresponding to $EN0$ into the last Karnaugh map.
- Then we minimize each of the 3 functions (for EN2, EN1 and EN0), both for minimal cost approach, and for minimal risk approach
- **Please do these steps as an exercise.**

Outline

Example 2: a modulo 6 binary count-down counter

Design a clocked synchronous state machine which counts cyclically modulo 6 in binary descending order. Obtain the excitation equations in the following cases:

(a) *for an implementation with D flip-flops, minimal cost approach*
(b) *for an implementation with D flip-flops, minimal risk approach*
(c) *for an implementation with T flip-flops with enable, minimal cost approach*
(d) *for an implementation with T flip-flops with enable, minimal risk approach*

Solution

- It is a modulo 6 counter, it means that we have 6 states, so we need 3 state variables.
- The initial state is state 5, encoded 101, so we have to initialize the state machine with this state at power-up and/or reset. This is obtained by connecting the general RESET line to the asynchronous Set input of the flip-flops Q2 and Q0, and to the asynchronous Reset input of the flip-flop Q1.
- The states are the decimal numbers from 5 down to 0 (the binary numbers from 101 down to 000).
- The counting sequence is $5 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 1 \rightarrow 0 \rightarrow 5$ and so on.
- Since the state coding is not necessary in this case, we can go directly to the transition table (table 9).
- When we use D flip-flops for implementation, the transition table is also the excitation table.

Excitation table for implementation with D flip-flops

Nr.	**Q2**	**Q1**	**Q0**	**Q2***	**Q1***	**Q0***
0	0	0	0	1	0	1
1	0	0	1	0	0	0
2	0	1	0	0	0	1
3	0	1	1	0	1	0
4	1	0	0	0	1	1
5	**1**	**0**	**1**	1	0	0
6	1	1	0	d/1	d/0	d/1
7	1	1	1	d/1	d/0	d/1
				D2	D1	D0

Table 9: Transition and excitation table for the count-down modulo 6 counter, including the states 6 and 7

- In the table the states are displayed starting from state 0, since in this way it is easier to copy the information in the Karnaugh maps
- State 5 is emphasized in the table
- It is equally well if we start from state 5.
- For the minimum risk approach, we consider that the “safe” state is state 5 (101).

The rest of the problem is left as an exercise.

Outline

Example 1: a modulo 6 binary count-up counter

Example 2: a modulo 6 binary count-down counter

Example 3: a zero's counter

Example 3: a modulo 5 0's counter

Design a clocked synchronous state machine which has an input X and an output Z. The output will be 1 if and only if the number of 0's (zero) received at the input X since reset is a multiple of 5, and the output will be 0 otherwise. Use D or T flip-flops for implementing the machine. Use either minimal risk or minimal cost approach.

Transition and output table for the modulo 5 0's counter, implementation with D flip-flops

				X=0			X=1			
Nr.	**Q2**	**Q1**	**Q0**	**Q2***	**Q1***	**Q0***	**Q2***	**Q1***	**Q0***	**Z**
0	0	0	0	0	0	1	0	0	0	1
1	0	0	1	0	1	0	0	0	1	0
2	0	1	0	0	1	1	0	1	0	0
3	0	1	1	1	0	0	0	1	1	0
4	1	0	0	0	0	0	1	0	0	0
5	1	0	1	d/0	d/0	d/0	d/0	d/0	d/0	d/0
6	1	1	0	d/0	d/0	d/0	d/0	d/0	d/0	d/0
7	1	1	1	d/0	d/0	d/0	d/0	d/0	d/0	d/0
				D2	D1	D0	D2	D1	D0	

Table 10: Transition and excitation table for the modulo 5 0's counter, for D flip-flops

Excitation equations

- We have obtain the excitation equations for D2, D1, D0 and Z, as functions of X, Q2, Q1 and Q0.
- This is a Moore machine, so the output Z depends only on the current state (Z will be a function of Q2, Q1 and Q0 only, it does not depend on X)
- For a Mealy machine, the outputs depend on both current state and the input variables.
- First, we have to obtain the excitation maps (i.e., the Karnaugh maps) for D2, D1, D0 and Z
- Then, to minimize the functions, resulting the excitation equations.
- For the Karnaugh maps we can consider X as the most significant input variable, or as the least significant input variable, either way is ok.

The rest of the problem is left as an exercise.